UNIVERSITY OF CALIFORNIA, DAVIS
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

COURSE: WATER RESOURCES SIMULATION (ECI 146)

INSTRUCTOR: ***Fabián A. Bombardelli***
(fabianbombardelli2@gmail.com, bmbrdll@yahoo.com, fabombardelli@ucdavis.edu)
OFFICE: 3105, Ghausi Hall (former Engineering III building)
Class: Tuesdays and Thursdays-2:10 to 3:30 PM (202 Wellman)
Computer lab: Wednesdays-11:00 AM to 11:50 AM, and 2:10 PM to 3 PM (Ghausi Hall 2030)

COMPUTER PROBLEM 2: A water-quality model for a lake -- eutrophication. Balance of nitrogen and phosphorus

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The LAKE model performs a mass balance of phosphorus and nitrogen in a lake, assuming that the lake can be schematized as a group of sub-compartments. The sub-compartments are the water column and the bed sediment.

There are inputs to, and "outwashes" from the lake, of phosphorus and nitrogen. Both phosphorus and nitrogen undergo sedimentation processes; sediment release from the bed (combining long-term processes of resuspension of particles and diffusion at the boundary); and demobilization in the sediment layer. In addition, nitrogen undergoes the process of denitrification, which is a chemical process. These processes and relations can be translated into the following set of *ordinary differential equations (ODEs)*:

$$\frac{d\,N_{wat}}{dt} = \frac{\left[\left(N_{load} - Denit\right) + N_{rel}\;N_{sed}\right]}{z} - \frac{1}{W_{res}}\;N_{wat}\;a - \frac{1}{z}\;SedRate\;N_{wat} \quad (1)$$

$$\frac{d\,N_{sed}}{dt} = SedRate\;N_{wat}\;\left(1 - N_{bound}\right) - N_{rel}\;N_{sed} \quad (2)$$

$$\frac{d\,P_{wat}}{dt} = \frac{\left[P_{load} + P_{rel}\;P_{sed}\right]}{z} - \frac{1}{W_{res}}\;P_{wat}\;a - \frac{1}{z}\;SedRate\;P_{wat} \quad (3)$$

$$\frac{d\,P_{sed}}{dt} = SedRate\;P_{wat}\;\left(1 - P_{bound}\right) - P_{rel}\;P_{sed} \quad (4)$$

The meaning of the above variables can be found in the hand-out of the LAKE model. The unknowns of the equations are: N_{wat}, P_{wat}, N_{sed}, and P_{sed}, with units of mg/l for the concentrations in the water, and g/m^2 for the sediment concentrations. Notice that the differential equations are linear in the unknown variables, as they can be rewritten as:

$$\frac{d\,N_{wat}}{dt} = A\;N_{sed} + B\;N_{wat} + C \tag{5}$$

$$\frac{d\,N_{sed}}{dt} = D\;N_{sed} + E\;N_{wat} \tag{6}$$

$$\frac{d\,P_{wat}}{dt} = F\;P_{sed} + G\;P_{wat} + H \tag{7}$$

$$\frac{d\,P_{sed}}{dt} = I\;P_{sed} + J\;P_{wat} \tag{8}$$

You are asked to:

1) Please, solve the above set of differential equations by the Euler method. Recall that the Euler method is the "forward method," so you need to evaluate the right-hand side at time j (see below). You can do this by developing a code in any language of your choice, or using a spreadsheet. Use the following values for the parameters: N_{load} = 25 g/m^2/year; N_{rel} = 0.9 1/year; N_{bound} = 0.1; $\boldsymbol{z}$ = 1.8 m; $SedRate$ = 30 m/year; W_{res} = 0.6 years; P_{load} = 1.6 g/m^2/year; P_{rel} = 0.8 1/year; P_{bound} = 0.05. The initial conditions for the computations are: $N_{wat}\big|_0$ = 4 mg/l; $N_{sed}\big|_0$ = 60 g/m^2; $P_{wat}\big|_0$ = 0.5 mg/l; $P_{sed}\big|_0$ = 15 g/m^2. Take $a = 1$. Perform the computations with a time step of 0.02 years and report the results at 5, 10, 15, 20 and 30 years (make a table with the results for those times). You should obtain four curves of the variables as a function of time. If one of the main variables turns negative, please check your computations. Please redo the computations with a time step of 0.01 years. Do you see differences with the previous run?

   The discrete version of the equations above is as follows:

$$\frac{N_{wat(j+1)} - N_{wat(j)}}{h} = A\;N_{sed(j)} + B\;N_{wat(j)} + C \tag{9}$$

$$\frac{N_{sed(j+1)} - N_{sed(j)}}{h} = D\;N_{sed(j)} + E\;N_{wat(j)} \tag{10}$$

$$\frac{P_{wat(j+1)} - P_{wat(j)}}{h} = F\;P_{sed(j)} + G\;P_{wat(j)} + H \tag{11}$$

$$\frac{P_{sed(j+1)} - P_{sed(j)}}{h} = I\ P_{sed(j)} + J\ P_{wat(j)} \tag{12}$$

where *j* indicates time and *h* is the time step.

2) Please redo your computations with a time step of 0.04 years. See if the solution is *unstable* for this step. If not, keep increasing it until *your results do not make sense* or *wiggles appear*. Determine this value of time step beyond which the solution starts to present this kind of "problematic behavior."
3) Develop a sensitivity analysis of the model to the parameters. To accomplish that, vary for instance N_{load} and P_{load}, to see the response of the system to those variables. Also, vary the residence time and the sedimentation rate. Analyze the results in terms of what you expect to happen and what really happens in your computations. Vary any other variable that you consider appropriate.
4) Imagine you are being hired by an engineering company and they ask you to make recommendations to improve the water quality of the lake. What would you recommend? Please discuss alternatives on how to improve the water quality of the lake based on your results.

Presentation of results: Please plot the time evolution of the nitrogen and phosphorus in two graphs. For the results of section 1), please present the results at 0, 5, 10, 15, 20 and years in a table. For other sections, please use either a table or plots.

Remark: What you are solving herein is for the values of nitrogen and phosphorus *as functions of time*; you are NOT iterating as in Computer Problem 1. Please keep this important difference in mind.

The grade in this project will be largely influenced by the discussion of the results you produce.